Correspondence

Linear Tapers in Rectangular Waveguides*

Recently, the special case of a linear double taper in rectangular waveguide propagating the TE_{10} mode in vacuum dielectric was examined.[1] Approximate expressions for the reflection coefficient and voltage standing-wave ratio as functions of the taper dimensions and free space wavelength were derived and experimentally verified. This correspondence generalizes the equations to be applicable for waveguides filled with dielectrics of arbitrary relative permittivity κ. As a matter of convenience, equations are numbered to correspond with similar equations in the referenced paper.

The approximate expression for the reflection coefficient is

$$\Gamma = \frac{1}{4\gamma_0}\left(\frac{d}{dx}\ln Z\right)_0 - \frac{1}{4\gamma_1}\left(\frac{d}{dx}\ln Z\right)_1 \cdot \exp\left[-2\int_0^L \gamma dx\right], \quad (6)$$

where γ is the propagation constant, Z is the characteristic impedance and L is the physical length of the taper. The subscripts 0 and 1 refer to conditions at the initial and terminal ends of the taper, respectively.

Consider a linear taper of length L connecting rectangular waveguides of impedance Z_0 and Z_1. In the taper section, the width and height of the guide are linear functions of the position:

$$a = a(x) = a_0 + \frac{a_1 - a_0}{L}x,$$

$$b = b(x) = b_0 + \frac{b_1 - b_0}{L}x.$$

To interpret (6) in terms of the TE_{10} mode in rectangular waveguide, the integrated characteristic impedance defined on a voltage-current basis is used. Let

$$Z = \frac{\pi\eta_0}{2}\,\frac{b}{a\sqrt{\kappa - (\lambda/2a)^2}} \quad (7)$$

and

$$\gamma = i\frac{2\pi}{\lambda_g} = i\frac{2\pi}{\lambda}\sqrt{\kappa - (\lambda/2a)^2}, \quad (8)$$

where η_0 is the impedance of free space, κ is the relative permittivity (dielectric constant) of the medium within the waveguide, λ is the free-space wavelength and λ_g is the guide wavelength. The logarithmic derivative in the taper is then found to be

$$\frac{d}{dx}\ln Z = \frac{1}{L}\left[\frac{b_1 - b_0}{b} - \frac{a_1 - a_0}{a}\left(\frac{\kappa}{\kappa - (\lambda/2a)^2}\right)\right]. \quad (9)$$

Substitution of (8) and (9) into (6) yields the following expression for the reflection coefficient:

$$\Gamma = \frac{i}{8\pi L/\lambda}\left[K_1 \exp(-i4\pi l) - K_0\right], \quad (11)$$

where

$$K_0 = \frac{\frac{b_1 - b_0}{b_0} - \frac{a_1 - a_0}{a_0}\left(\frac{\kappa}{\kappa - (\lambda/2a_0)^2}\right)}{[\kappa - (\lambda/2a_0)^2]^{1/2}} \quad (12)$$

$$K_1 = \frac{\frac{b_1 - b_0}{b_1} - \frac{a_1 - a_0}{a_1}\left(\frac{\kappa}{\kappa - (\lambda/2a_1)^2}\right)}{[\kappa - (\lambda/2a_1)^2]^{1/2}}, \quad (13)$$

and

$$l = \int_0^L \frac{dx}{\lambda_g} = \frac{1}{\lambda}\int_0^L \sqrt{\kappa - (\lambda/2a)^2}\, dx. \quad (14)$$

Eq. (14) may be integrated with the result that

$$l = \frac{L}{2(a_1 - a_0)}\left[\frac{2a_1}{\lambda_{g_1}} - \frac{2a_0}{\lambda_{g_0}} + \tan^{-1}\frac{2a_0}{\lambda_{g_0}} - \tan^{-1}\frac{2a_1}{\lambda_{g_1}}\right], \quad (15)$$

where

$$\lambda_{g_0} = \lambda/\sqrt{\kappa - (\lambda/2a_0)^2}$$

and

$$\lambda_{g_1} = \lambda/\sqrt{\kappa - (\lambda/2a_1)^2}.$$

The absolute magnitude of the reflection coefficient is

$$|\Gamma| = \frac{1}{L/\lambda}\left[\frac{K_0^2 + K_1^2}{64\pi^2} - \frac{K_0 K_1}{32\pi^2}\cos(4\pi l)\right]^{1/2}. \quad (16)$$

Using

$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|}, \quad (17)$$

the dominant mode voltage standing-wave ratio (VSWR) can be calculated as a function of frequency and taper length for a linear taper connecting two specified rectangular waveguides. The above equations reduce to those of the referenced paper if κ is replaced by unity.

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[1] R. C. Johnson, "Design of linear double tapers in rectangular waveguide," IRE Transactions on Microwave Theory and Techniques, Vol. MTT-7, pp. 374–378; July, 1959. (Corrections: Vol. MTT-8, p. 458; July, 1960.)

The Unloaded Q of a YIG Resonator from X-Band to 4 Millimeters*

Carter and Flammer[1] have reported measurements at lower microwave frequencies of Q_u, the unloaded Q of a single crystal YIG sphere treated as a resonator. In their paper a comparison of the measured and theoretical Q_u, the latter computed on the basis of a constant relaxation time, yielded good agreement only in the 2–5 kMc range.

In this paper we report results of the variation of Q_u with frequencies from 9.5 to 67.8 kMc. Our measurements indicate that in this range Q_u is approximately constant.

Q_u may be found theoretically by solving the equation of motion of the freely processing magnetization in a saturated ferrimagnet. This equation, with Landau-Lifshitz damping, is

$$\boldsymbol{M} = \gamma(\boldsymbol{M} \times \boldsymbol{H}) + \frac{\gamma_\alpha}{M_s}\boldsymbol{M} \times (\boldsymbol{M} \times \boldsymbol{H}). \quad (1)$$

We consider the general ellipsoid with principal axes parallel to the x-y-z directions, and with the applied field H_0 in the z direction. Using the MKS rationalized system of units with $\boldsymbol{B} = \mu_0\boldsymbol{H} + \boldsymbol{M}$, the differential equation for m_x is

$$\ddot{m}_x + \alpha|\gamma|\left[2H_0 + [N_x + N_y - 2N_z]\frac{M_s}{\mu_0}\right]\dot{m}_x + \omega_0^2 m_x = 0 \quad (2)$$

where

$$\omega_0 = |\gamma|\left[\left(H_0 + (N_x - N_z)\frac{M_s}{\mu_0}\right) \cdot \left(H_0 + (N_y - N_z)\frac{M_s}{\mu_0}\right)\right]^{1/2}.$$

Q_u is therefore given by[2]

$$Q_u = \frac{\omega_0/|\gamma|}{\alpha\left[\frac{4\omega_0^2}{\gamma^2} + \left(\frac{M_s}{\mu_0}(N_x - N_y)\right)^2\right]^{1/2}}. \quad (3)$$

For a sphere, where $\omega_0 = \gamma H_0$,

$$Q_u = \frac{1}{2\alpha}. \quad (4)$$

To compute Q_u from the equation of motion with Bloch-Bloembergen damping,[3] we use

$$\dot{M}_{x,y} = \gamma(\boldsymbol{M} \times \boldsymbol{H})_{x,y} - \frac{M_{x,y}}{T_2}. \quad (5)$$

* Received by the PGMTT, January 16, 1961.

[1] P. S. Carter and C. Flammer, "Unloaded Q of single crystal yttrium-iron garnet resonator as a function of frequency," IRE Transactions on Microwave Theory and Techniques, vol. MTT-8, pp. 570–571; September, 1960.

[2] A. R. Von Hippel, "Dielectrics and Waves," J. Wiley and Sons, Inc., New York, N. Y., pp. 101–102; 1954.

[3] N. Bloembergen, "On the ferromagnetic resonance in nickel and supermalloy," *Phys. Rev.*, vol. 78, pp. 572–580; June, 1950.